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Power corrections to the process $\gamma\gamma^* \to \pi\pi$ in the light-cone sum rules approach

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Abstract. We applied QCD light-cone sum rules to estimate power corrections to the helicity-conserving amplitude in the process $\gamma^*\gamma \to \pi\pi$. We found that above $Q^2 \sim 4\,\mathrm{GeV}^2$ power corrections are numerically small and the twist-2 part dominates.The amplitude can be reliably calculated in this region using models of 2π distribution amplitudes as input. We found that the magnitude of the NLO corrections depends rather strongly on the normalization of the gluonic distribution amplitude.

1 Introduction

Hadron production in the reaction $\gamma^*\gamma \to$ hadron(s) has been a subject of considerable interest for a long time, both from the experimental [1,2] and theoretical [3–5] points of view. The key role in the QCD description of such processes is played by the QCD factorization theorem. For example, QCD factorization has been successfully applied to the reaction $\gamma^*\gamma \to \pi^0$ [4,5]. The $F^{\gamma\pi}(Q^2)$ form-factor data obtained by the CELLO and CLEO collaborations are in a good agreement with the available QCD analysis; see for example [6–8].

Recently it has been proposed [9,10] to investigate a similar process, $\gamma^*\gamma \to \pi\pi$, when the two-pion state has a small invariant mass. It has been argued that QCD factorization applies to this case as well [11]. The resulting amplitude depends on new non-perturbative objects, the so-called two-pion distribution amplitudes (2πDAs). They are given by matrix elements of twist-2 QCD string operators between the vacuum and the two-pion state [10,12]. Moreover, 2πDAs can be related by the crossing symmetry to skewed parton distributions [13,14] which recently have been subject of considerable interest.

Furthermore, in a recent paper [15] it has been argued that experimental studies of 2π production cross-section are possible with existing e^+e^- facilities.

Formally, the dominance of the leading-twist amplitude is guaranteed only at a very large Q^2. For the process $\gamma^*\gamma \to \pi\pi$ the bulk of the twist-2 amplitude arises from the handbag diagram. In analogy with the $\gamma^*\gamma \to \pi^0$ reaction one expects that the leading-twist contribution dominates the amplitude even for moderate values of $Q^2 \sim 4$–$10\,\mathrm{GeV}^2$. For lower values of Q^2, the power-suppressed corrections are certainly important. Note that preliminary estimates show that most of the $\gamma^*\gamma \to \pi\pi$ events which have been seen in the CLEO data are in the region $Q^2 \sim 1$–$5\,\mathrm{GeV}^2$ [2,16]. In this region a reliable estimate of the amplitude including the power-suppressed contributions is crucial.

From the theoretical point of view problems encountered in an analysis of power-suppressed contributions to two-pion and one-pion production amplitudes are very similar. Terms suppressed as $1/Q^2$ can arise from different space-time configurations. The production of states with more than two partons by interaction of two electromagnetic currents at small transverse distances can be accounted for by the standard operator product expansion (OPE) technique. However, as the second photon is real, there is yet another configuration which results in a power-suppressed correction. The real photon can turn into hadrons long before the interaction with the virtual one. It occurs at large transverse distances between two electromagnetic currents. Such a non-factorizable term is known in the literature as the "end-point" or "soft" contribution.

In this paper we use the light-cone sum rule (LCSR) method to evaluate the $\gamma^*\gamma \to \pi\pi$ amplitude, including the power-suppressed contributions. The main advantage of this technique is that it allows one to take into account both factorizable and non-factorizable corrections. Recently, LCSRs were successfully applied to describe different pion form factors [7,17,18] in a Q^2 range from 1 to $\sim 10\,\mathrm{GeV}^2$.

This paper is organized as follows. In the following section we present the definition of the $\gamma^*\gamma \to \pi\pi$ amplitude and set up the notation. In Sect. 3 models of 2π distribution amplitudes are introduced. Sections 4 and 5 are devoted to a discussion of the LCSRs to the LO and NLO accuracy, respectively. In Sect. 6 we present a numerical analysis. Finally, we summarize our article. The appendix contains definitions of the NLO Wilson coefficient functions.

2 General definitions

The kinematics of the reaction $\gamma^*(q)\gamma(q') \to \pi(k_1)\pi(k_2)$ can conveniently be described in terms of a pair of light-like vectors p, z which obey

$$p^2 = z^2 = 0, \quad p \cdot z \neq 0, \tag{1}$$

and define longitudinal directions. Here $p \cdot z = p_\mu z^\mu$. Let P and k denote the total and relative momenta of the π meson pair, respectively,

$$\begin{aligned} P^2 &= (k_1 + k_2)^2 = W^2, \\ k^2 &= (k_1 - k_2)^2 = 4m_\pi^2 - W^2, \\ P \cdot k &= 0. \end{aligned} \tag{2}$$

The initial and final states momenta can be decomposed as

$$\begin{aligned} &q = p - \frac{Q^2}{2(p\cdot z)} z, \quad q^2 = -Q^2 \quad q' = \frac{Q^2 + W^2}{2(p\cdot z)} z, \\ &q'^2 = 0, \quad P = q + q' = p + \frac{W^2}{2(p\cdot z)} z, \quad P^2 = W^2, \\ &k = \xi p - \frac{\xi W^2}{2(p\cdot z)} z + k_\perp. \end{aligned} \tag{3}$$

The longitudinal momentum distribution of the pions is described by the variable $\xi = (k\cdot z)/(p\cdot z)$. Alternatively,

$$\xi = \beta \cos\theta_{\mathrm{cm}},$$

where θ_{cm} is the polar angle of the pion momentum in the CM frame with respect to the direction of the total momentum P, and β is the velocity of produced pions in the center-of-mass frame

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{W^2}}.$$

The amplitude of hard photo-production of two pions is defined by the following matrix element between the vacuum and the two-pion state:

$$\begin{aligned} T^{\mu\nu} &= \mathrm{i} \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}x\cdot\bar{q}} \langle 2\pi(P,k)| T J^\mu(x/2) J^\nu(-x/2)|0\rangle, \\ \bar{q} &= \frac{1}{2}(q - q'), \end{aligned} \tag{4}$$

where $J^\mu(x)$ denotes the quark electromagnetic current. Hard photo-production corresponds to the limit $Q^2 \gg W^2 \geq \Lambda^2_{\mathrm{QCD}}$ where the amplitude (4) can be represented as an expansion in terms of powers of $1/Q$. According to the factorization theorem the leading-twist term in the expansion can be written as a convolution of hard and soft blocks. The coefficient functions can be calculated from appropriate partonic subprocesses $\gamma^* + \gamma \to \bar{q} + q$ or $\gamma^* + \gamma \to g + g$.

According to the analysis of [19] to the leading-twist accuracy the amplitude $T^{\mu\nu}$ is a sum of two terms

$$\begin{aligned} T^{\mu\nu}(q, q', P, k) &= \frac{\mathrm{i}}{2}(-g^{\mu\nu})_{\mathrm{T}} T_0^{\gamma\pi\pi}(q, q', P, k) \\ &+ \frac{\mathrm{i}}{2} \frac{k_\perp^{(\mu} k_\perp^{\nu)}}{W^2} T_2^{\gamma\pi\pi}(q, q', P, k), \end{aligned} \tag{5}$$

where

$$(-g^{\mu\nu})_{\mathrm{T}} = \left(\frac{p^\mu z^\nu + p^\nu z^\mu}{p\cdot z} - g^{\mu\nu} \right)$$

is the metric tensor in the transverse space, and $k_\perp^{(\mu} k_\perp^{\nu)}$ denotes the traceless, symmetric tensor product of the relative transverse momenta (27).

The leading-order, leading-twist amplitude $T_0^{\gamma\pi\pi}$ describes scattering of two photons with equal helicities, related by crossing to the photon helicity-conserving DVCS on a pion. At the NLO there is a new contribution $T_2^{\gamma\pi\pi}$ from collisions of photons with opposite helicities, related to the photon helicity-flip contribution to the DVCS [20]. In terms of the operator product expansion the latter amplitude singles out a twist-2 tensor gluon operator which cannot be studied in deep-inelastic scattering (DIS) on a pion (or nucleon) target [19].

Note that as the amplitude $T^{\mu\nu}$ is dimensionless, the twist-2 amplitude $T_0^{\gamma\pi\pi}$ depends on Q^2 only logarithmically through the running coupling and QCD evolution effects. To see this it is convenient to develop appropriate power counting in an infinite momentum frame. For definiteness we assume that the pion pair is moving in the positive $\hat{z}$ direction and p^+ and z^- are the only nonzero component of p and z, respectively. Then the infinite momentum frame can be understood as $p^+ \sim Q \to \infty$ with a fixed $(p\cdot z) \sim 1$. From (3) it follows that in this frame $(k\cdot z) \sim 1$ and $k_\perp \sim Q^0$. This determines the power counting in Q for all twist-2 amplitudes and from (5) we find that $T_0^{\gamma\pi\pi}$ is $O(1)$ as far as powers of Q are concerned.

In this paper we consider power corrections to the amplitude $T_0^{\gamma\pi\pi}$ only. This is expected to be the most important term numerically as it appears already at the Born level. To the NLO accuracy one has [19]

$$\begin{aligned} T_0^{\gamma\pi\pi} = T_0^{\mathrm{pert}} &= \left(\sum e_q^2\right) \int_0^1 \mathrm{d}u \Phi^Q(u, \xi, W^2) \\ &\times \left[C_q^0(u) + \tfrac{\alpha_{\mathrm{S}}(Q^2)}{4\pi} C_q^1(u) \right] \\ &- \left(\sum e_q^2\right) \int_0^1 \mathrm{d}u \Phi^G(u, \xi, W^2) \\ &\times \left[\tfrac{\alpha_{\mathrm{S}}(Q^2)}{4\pi} C_g^1(u) \right]. \end{aligned} \tag{6}$$

The coefficient functions C_q^0, C_q^1, C_g^1 can be found in [19]. The quark and gluon 2πDAs are defined as matrix elements of the light-cone string operators:

$$\langle \pi\pi(P,k) \left| \frac{1}{N_f} \sum_q \bar{q}(z)\hat{z}q(-z) \right| 0\rangle = (p\cdot z)$$

$$\times \int_0^1 \mathrm{d}u \Phi^Q(u,\xi,W^2) \mathrm{e}^{\mathrm{i}(2u-1)(p\cdot z)}, \quad (7)$$

$$\langle \pi\pi(P,k) \left| z_\mu z_\nu G^\mu{}_\alpha(z) G^{\alpha\nu}(-z) \right| 0 \rangle = (p\cdot z)^2 \times \int_0^1 \mathrm{d}u \Phi^G(u,\xi,W^2) \mathrm{e}^{\mathrm{i}(2u-1)(p\cdot z)}. \quad (8)$$

Note that the distribution amplitudes $\Phi^Q(u,\xi,W^2)$ and $\Phi^G(u,\xi,W^2)$ depend also on a factorization scale μ.

3 Models for two-pion distribution amplitudes

In this section we describe briefly the main properties of the distribution amplitudes introduced in (7) and (8) and discuss a model which has been used to obtain estimates for the magnitude of power-suppressed corrections.

Due to the positive C-parity of the pion pair, 2π distribution amplitudes have the following symmetry properties:

$$\Phi^Q(u,\xi,W^2) = -\Phi^Q(1-u,\xi,W^2) = \Phi^Q(u,-\xi,W^2),$$
$$\Phi^G(u,\xi,W^2) = \Phi^G(1-u,\xi,W^2) = \Phi^G(u,-\xi,W^2). \quad (9)$$

The factorization scale dependence is governed by the ERBL evolution equations [21,22]. As is well known, in the leading logarithmic approximation their solution has the form of an expansion in terms of Gegenbauer polynomials:

$$\Phi^Q(u,\xi,W^2|\mu) = 6u(1-u) \times \sum_{\substack{n=1\\ \text{odd}}}^{\infty} B_n(\xi,W^2|\mu) C_n^{3/2}(2u-1), \quad (10)$$

$$\Phi^G(u,\xi,W^2|\mu) = 30u^2(1-u)^2 \times \sum_{\substack{n=0\\ \text{even}}}^{\infty} A_n(\xi,W^2|\mu) C_n^{5/2}(2u-1). \quad (11)$$

The coefficients B_n and A_n mix under evolution.

In the next step one expands both distribution amplitudes in the partial waves of the final two-pion system [12]. As a result one introduces an expansion of $B_n(\xi,W^2|\mu)$ and $A_n(\xi,W^2|\mu)$ in terms of Legendre polynomials $P_l(\xi)$ [12,19]:

$$B_n(\xi,W^2|\mu) = \sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}(W^2|\mu) P_l(\xi),$$
$$A_n(\xi,W^2|\mu) = \sum_{\substack{l=0\\ \text{even}}}^{n+2} A^G_{nl}(W^2|\mu) P_l(\xi). \quad (12)$$

Additional constraints on 2π distribution amplitudes are provided by soft pion theorems [12]:

$$\Phi^Q(u,\xi=1,W^2=0) = \Phi^Q(u,\xi=-1,W^2=0) = 0,$$
$$\Phi^G(u,\xi=1,W^2=0) = \Phi^G(u,\xi=-1,W^2=0) = 0. \quad (13)$$

Finally, crossing symmetry allows one to relate moments of distribution amplitudes to forward matrix elements of twist-2 operators which determine moments of pion quark and gluon structure functions; see [12] for details. One finds, e.g.

$$\int_0^1 \mathrm{d}u(2u-1)\Phi^Q(u,\xi=1,W^2=0) = -\frac{1}{N_f} M^Q(1-\xi^2)$$
$$\int_0^1 \mathrm{d}u \Phi^G(u,\xi=1,W^2=0) = -\frac{1}{2} M^G(1-\xi^2), \quad (14)$$

where M^Q and M^G are the momentum fractions carried by quarks and gluons in a pion:

$$M^Q(\mu) = \int_0^1 \mathrm{d}u u \sum_q (q_\pi(u,\mu) + \bar{q}_\pi(u,\mu)),$$
$$M^G(\mu) = \int_0^1 \mathrm{d}u u g_\pi(u,\mu). \quad (15)$$

At asymptotically large $\mu^2 \to \infty$ only the lowest terms in (10), (11) contribute. Combining constraints (14) with (12) one easily finds

$$\Phi^G_{\mathrm{as}}(u,\zeta,W^2=0) = -15u^2(1-u)^2 M^G_{\mathrm{as}}(1-\xi^2),$$
$$\Phi^Q_{\mathrm{as}}(u,\zeta,W^2=0) = -30u(1-u)\ (2u-1)\frac{1}{N_f} \times M^Q_{\mathrm{as}}(1-\xi^2). \quad (16)$$

where

$$M^Q_{\mathrm{as}} = \frac{N_f}{N_f+4C_F}, \quad M^G_{\mathrm{as}} = \frac{4C_F}{N_f+4C_F}. \quad (17)$$

For $W^2 \neq 0$ one writes

$$\Phi^G_{\mathrm{as}}(u,\xi,W^2) = -15u^2(1-u)^2 M^G_{\mathrm{as}} B(\xi,W^2), \quad (18)$$
$$\Phi^Q_{\mathrm{as}}(u,\xi,W^2) = -30u(1-u)(2u-1)\frac{1}{N_f} M^Q_{\mathrm{as}} B(\xi,W^2).$$

The function $B(\xi,W^2)$ is related to coefficients B_1 and A_0 from (12)

$$B(\xi,W^2) = -\frac{2}{M^G_{\mathrm{as}}} A_0(\xi,W^2|\mu^2=\infty)$$
$$= -\frac{3}{5}\frac{N_f}{M^Q_{\mathrm{as}}} B_1(\xi,W^2|\mu^2=\infty). \quad (19)$$

In the limit $W^2 \to 0$ one finds from (14) that $B(\xi,W^2=0) = (1-\xi^2)$.

In numerical calculations presented in this paper we have used a model which retains a simple analytical form of the asymptotic distributions amplitudes (18), but incorporates nontrivial information about the pion structure at a scale of the order of a few GeV^2 [15]. Assuming dominance of the lowest conformal wave one finds

$$\Phi^G(u,\xi,W^2) = -15u^2(1-u)^2 M^G(\mu^2) B(\xi,W),$$
$$\Phi^Q(u,\xi,W^2) = -30u(1-u)(2u-1)\frac{1}{N_f} \times M^Q(\mu^2) B(\xi,W). \quad (20)$$

As at present a little is known about 2πDAs, the main motivation beyond the model (20) is that it incorporates all constraints arising from crossing symmetry and soft pion theorems and has a simple form which makes its treatment in numerical calculations easy. Note that dominance of the lowest conformal wave seems to be phenomenologically justified in the case of the single pion DA.

This model differs from the asymptotic DA (18) only in values of momentum fractions $M^G(\mu^2)$ and $M^Q(\mu^2)$. Their scale dependence is given by

$$M^Q(Q^2) = M^Q_{\mathrm{as}}\left(1 + L(Q^2)R(\mu^2)\right),$$
$$R(\mu^2) = \frac{M^Q(\mu^2) - M^Q_{\mathrm{as}}}{M^Q_{\mathrm{as}}}, \tag{21}$$

where L is the usual evolution factor:

$$L(Q^2) = \left(\frac{\alpha_{\mathrm{S}}(Q^2)}{\alpha_{\mathrm{S}}(\mu^2)}\right)^{\gamma_+/b_0}, \quad \gamma_+ = \frac{2}{3}(N_f + 4C_F),$$
$$b_0 = \frac{11}{3} - \frac{2}{3}N_f. \tag{22}$$

Obviously, $M^G(Q^2) = 1 - M^Q(Q^2)$.

An explicit expression for $B(\xi, W)$ can be obtained using the Watson theorem [12]. In the calculations considered here $B(\xi, W)$ enters as a Q^2-independent factor and therefore its explicit functional form is not important for considerations of power-suppressed corrections to the amplitude. To remove this factor from numerical calculations we will consider the Q^2-dependence of the ratio $T_0^{\gamma\pi\pi}(Q^2, \xi, W^2)/T_0^{\mathrm{as}}$, where

$$\begin{aligned} T_0^{\mathrm{as}} &= T_0^{\gamma\pi\pi}(Q^2 = \infty, \xi, W^2) \\ &= \sum_q e_q^2 \int_0^1 \mathrm{d}u \frac{2u\Phi^Q_{\mathrm{as}}(u, \xi, W^2)}{1-u} \\ &= -\frac{15}{14}\sum_q e_q^2 B(\xi, W^2) \end{aligned} \tag{23}$$

is the asymptotic value of the amplitude.

In Fig. 1 we compare this ratio for the NLO amplitude $T_0^{\mathrm{pert}}(Q^2, \xi, W^2)$ for different models of 2π distribution amplitudes: the asymptotic (18) and its minimal extension (20). Note that there is an ambiguity due to the scale dependence of the NLO amplitude. In order to estimate this uncertainty we make the following choice for the scale μ:

$$\mu^2 = \kappa Q^2 + M^2, \tag{24}$$

with $M^2 = 1\,\mathrm{GeV}^2$ and a parameter κ which will be varied in the interval $1/5 \le \kappa \le 1$. Such a choice is motivated by the observation [18] that a typical virtuality of a propagator in a perturbative, exclusive amplitude is given by a weighted sum of the hard scale Q^2 and the infrared cutoff. In the LCSR approach the latter role is played by the Borel mass M^2; see the next section. Evaluating the NLO amplitude (6) we have neglected numerically small NLO corrections [23–25] to the evolution of the distribution amplitudes in the $\overline{\mathrm{MS}}$ scheme.

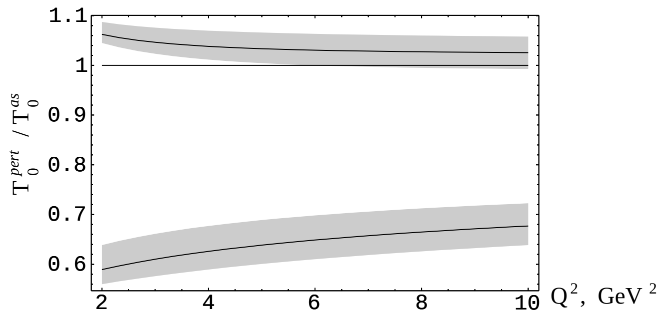

Fig. 1. NLO results for the $T_0^{\mathrm{pert}}(Q^2)/T_0^{\mathrm{as}}$ as a function of Q^2. The upper plot corresponds to the mimimal model with $M^Q(1\,\mathrm{GeV}^2) = 0.60$ and the lower one to the asymptotic choice $M^Q = 0.43$. The scale is taken as $\mu^2 = \kappa Q^2 + M^2$, $M^2 = 1\,\mathrm{GeV}^2$. Gray bands show the variation of the NLO prediction when κ is varied between $1/5$ and 1. Solid lines correspond to $\kappa = 2/5$

One finds that in the model based on asymptotic distribution amplitudes the NLO correction is numerically much larger than in the second model, where amplitudes have the asymptotic form but a different, scale-dependent normalization. This can be understood by observing that the gluon distribution amplitude gives the main contribution to the NLO correction. At Q^2 of order of few GeV^2 its normalization in (20), based on the momentum fraction carried by gluons in a pion according to the GRV parameterization [26] is much smaller than the asymptotic one. The strong sensitivity of the NLO corrections to the gluon distribution amplitude is an interesting feature of the process considered here. Therefore we have to revise one of our conclusions from [19], about the size of the NLO correction. Contrary to our previous claim, its magnitude turns out to be rather model dependent, and therefore it is difficult to make a trustworthy prediction unless the distribution amplitudes which enter the NLO correction, in particular the gluon one, are sufficiently constrained.

4 Light-cone sum rules method: the LO approximation

In this section we discuss the light-cone sum rule (LCSR) for the amplitude $T_0^{\gamma\pi\pi}$. Our procedure closely follows the investigation of the photon–pion transition form factor $F_{\gamma\pi}(Q^2)$ in [17].

The first step to obtain a LCSR for the amplitude with one real photon $T_0^{\gamma\pi\pi}$ is to consider an amplitude $T_0^{\gamma^*\pi\pi}$ where both photons are off-shell and have large virtualities:

$$\gamma^*(q) + \gamma^*(q') \to 2\pi(P, k), \quad Q^2, Q'^2 \gg \Lambda^2_{\mathrm{QCD}}, \tag{25}$$

and find its dispersion representation in the variable q'^2.

In the general kinematics momenta q and q' can be represented in terms of vectors p and z as

$$q = -\frac{Q^2}{2\sigma(p\cdot z)}p + \sigma z, \quad q^2 = -Q^2,$$

$$q' = -\frac{Q'^2}{2\alpha(p\cdot z)}p + \alpha z, \quad q'^2 = -Q'^2. \tag{26}$$

The coefficients α and σ are functions of the kinematical invariants. Using momentum conservation one finds

$$\alpha = \frac{W^2}{2(p\cdot z)} - \sigma, \quad \sigma = \frac{1}{2(p\cdot z)}(q\cdot(q+q') - \sqrt{X}),$$
$$X = (q\cdot q')^2 - q^2 q'^2. \tag{27}$$

With the help of the factorization theorem one can write $T_0^{\gamma^*\pi\pi}$ as a convolution of hard and soft blocks. The virtuality q'^2 enters the hard part only, where we neglect W^2 as compared with Q^2 and Q'^2.

With $q'^2 \neq 0$ the two-photon amplitude admits naturally a richer Lorentz structure than the original amplitude with one photon on-shell. In addition, by splitting $T^{\mu\nu}$ into Lorentz tensors and invariant coefficient functions it is advantageous to avoid kinematical constraints for the latter. Constraints imposed on coefficient functions result in constraints on the form of their dispersion representation.

To this end we rewrite the transverse metric tensor in terms of momenta q and q' [3]:

$$R^{\mu\nu}(q,q') = (-g^{\mu\nu})_{\mathrm{T}} = -g^{\mu\nu} + \frac{1}{X}(q\cdot q'(q^\mu q'^\nu + q^\nu q'^\mu)$$
$$- q^2 q'^\mu q'^\nu - q'^2 q^\mu q^\nu), \tag{28}$$

with X defined in (27), and introduce a variable ω

$$\omega = \frac{2P\cdot q}{q^2} = \frac{W^2}{Q^2} + \frac{q^2 - q'^2}{q^2} \simeq \frac{q^2 - q'^2}{q^2}. \tag{29}$$

With these definitions it is convenient to introduce $T_0^{\gamma^*\pi\pi}$ by

$$T^{\mu\nu} = \mathrm{i}\frac{\omega^2}{2}R^{\mu\nu}(q,q')T_0^{\gamma^*\pi\pi}(Q,q',\xi,W) + \ldots, \tag{30}$$

where the ellipsis denotes other possible Lorentz structures. Now, in the limit $-q'^2 \to 0$ ($\omega \to 1$) $T_0^{\gamma^*\pi\pi}$, defined here, goes smoothly into $T_0^{\gamma\pi\pi}$ given by (6). On the other hand, the factor ω^2 cancels the singularity present in the tensor $R^{\mu\nu}$ as $q^2 \to q'^2, W^2 \to 0$ ($\omega \to 0$) due to the $1/X$ term, hence no constraints have to be imposed on $T_0^{\gamma^*\pi\pi}$.

To the LO accuracy and keeping $W^2 = 0$ in the hard block the amplitude $T_0^{\gamma^*\pi\pi}(Q,q',\xi,W)$ is the only one which appears in $T^{\mu\nu}$ and one obtains

$$T_0^{\gamma^*\pi\pi}(Q,q',\xi,W)$$
$$= \left(\sum_q e_q^2\right)\int_0^1 \mathrm{d}x \Phi^Q(x,\xi,W^2)\frac{2x}{1-x\omega}$$
$$= \left(\sum_q e_q^2\right)\int_0^1 \mathrm{d}x \Phi^Q(x,\xi,W^2)\frac{2xQ^2}{(1-x)Q^2 - xq'^2}. \tag{31}$$

Note that the above expression depends on W through the two-pion distribution amplitude (7).

The LO result can easily be converted into a dispersion integral over q'^2 with $s = \bar{x}Q^2/x$ being the mass of the intermediate state. Using the symmetry properties of the quark DA one finds

$$T_0^{\gamma^*\pi\pi}(Q,q',\xi,W) = \int_0^\infty \mathrm{d}s\frac{\rho_0(Q,s,\xi,W)}{s-q'^2}, \tag{32}$$

where

$$\rho_0(Q,s,\xi,W) = 2\left(\sum_q e_q^2\right)\int_0^1 \mathrm{d}x\delta\left(x - \frac{Q^2}{s+Q^2}\right)$$
$$\times x^2\Phi^Q(x,\xi,W^2). \tag{33}$$

The next step is to rewrite the dispersion relation in the q'^2 channel assuming that the spectral density can be approximated by contributions of low-lying hadron states ρ,ω and a continuum of higher-mass states with an effective threshold s_0:

$$T_0^{\gamma^*\pi\pi}(Q,q',\xi,W) = \sqrt{2}f_\rho\frac{T^{\rho\pi\pi}(Q,\xi,W)}{m_\rho^2 - q'^2}$$
$$+ \int_{s_0}^\infty \mathrm{d}s\frac{\rho^{\mathrm{cont}}(Q,s,\xi,W)}{s-q'^2}. \tag{34}$$

Here we have introduced the following notation for the matrix elements:

$$\langle\rho^0|J_\nu|0\rangle = \frac{1}{\sqrt{2}}f_\rho m_\rho \epsilon_\nu^*, \tag{35}$$
$$\langle\pi\pi|J^\mu|\rho^0\rangle = \frac{\mathrm{i}}{m_\rho}\frac{\omega^2}{2}R^{\mu\nu}(q,q')\epsilon_\nu T_0^{\rho\pi\pi}(Q,\xi,W) + \ldots$$

$\epsilon_\nu^*, \epsilon_\nu$ are the polarization vectors of the ρ meson. To include the ω meson we adopt the following approximate relations:

$$m_\omega \simeq m_\rho, \quad 3f_\omega \simeq f_\rho,$$
$$T_0^{\omega\pi\pi}(Q,\xi,W) \simeq 3T_0^{\rho\pi\pi}(Q,\xi,W), \tag{36}$$

which follow from the quark content of ρ and ω and from the isospin symmetry.

The right-hand side (RHS) of (34) involves two unknown functions: the form factor $T_0^{\rho\pi\pi}$ and the spectral density ρ^{cont}. Assuming quark–hadron duality one can estimate the continuum spectral density as

$$\rho^{\mathrm{cont}}(Q,s,\xi,W) = \theta(s>s_0)\rho_0(Q,s,\xi,W), \tag{37}$$

where $\rho_0(Q,s,\xi,W)$ is the spectral density given in (33). By keeping $-q'^2$ large and combining (37), (34) and (31) one obtains a LCSR for the form factor $T_0^{\rho\pi\pi}$:

$$\sqrt{2}f_\rho\frac{T_0^{\rho\pi\pi}(Q,\xi,W)}{m_\rho^2 - q'^2} = \int_0^{s_0}\mathrm{d}s\frac{\rho_0(Q,s,\xi,W)}{s-q'^2}. \tag{38}$$

After perfoming a Borel transformation in $-q'^2$ one finds (M is the Borel mass):

$$\sqrt{2}f_\rho T_0^{\rho\pi\pi}(Q,\xi,W)$$
$$= \int_0^{s_0}\mathrm{d}s\rho_0(Q,s,\xi,W)\mathrm{e}^{(m_\rho^2 - s)/M^2}. \tag{39}$$

Finally, substituting (37) and (39) into (34) results in

$$T_0^{\gamma^*\pi\pi}(Q,q'^2,\xi,W) = \frac{1}{m_\rho^2 - q'^2} \times \int_0^{s_0} \mathrm{d}s \rho_0(Q,s,\xi,W) \mathrm{e}^{(m_\rho^2 - s)/M^2} + \int_{s_0}^{\infty} \mathrm{d}s \frac{\rho_0(Q,s,\xi,W)}{s - q'^2}. \quad (40)$$

The above representation allows one to perform an analytical continuation to the point $q'^2 = 0$. In this way one arrives at a LCSR for the amplitude with one real photon:

$$T_0^{\gamma\pi\pi}(Q,\xi,W) = \frac{1}{m_\rho^2} \int_0^{s_0} \mathrm{d}s \rho_0(Q,s,\xi,W) \mathrm{e}^{(m_\rho^2 - s)/M^2} + \int_{s_0}^{\infty} \frac{\mathrm{d}s}{s} \rho_0(Q,s,\xi,W). \quad (41)$$

It is convenient to rewrite this formula going back to an integral over the fraction $x = Q^2/(s+Q^2)$. Introducing $x_0 = Q^2/(s_0+Q^2)$ and $\bar{x} \equiv 1-x$ one finds

$$T_0^{\gamma\pi\pi}(Q,\xi,W) = \frac{2Q^2}{m_\rho^2} \left(\sum_q e_q^2\right) \int_{x_0}^1 \mathrm{d}x \Phi^Q(x,\xi,W^2) \times \exp\left\{\frac{x m_\rho^2 - \bar{x} Q^2}{x M^2}\right\} + \left(\sum_q e_q^2\right) \int_0^{x_0} \mathrm{d}x \frac{2x}{1-x} \Phi^Q(x,\xi,W^2). \quad (42)$$

Consider now the limit $Q^2 \to \infty$. In this limit $x_0 = 1 - s_0/Q^2 + O(1/Q^4)$. The integration region in the first term in the RHS (42) shrinks to the point $x = 1$ and one obtains

$$\frac{2Q^2}{m_\rho^2} \int_{x_0}^1 \mathrm{d}x \Phi^Q(x,\xi,W^2) \exp\left\{\frac{x m_\rho^2 - \bar{x} Q^2}{x M^2}\right\} = \frac{2s_0^2}{Q^2 m_\rho^2} \Phi_x^Q(1,\xi,W^2) \int_0^1 \mathrm{d}x x \mathrm{e}^{(m_\rho^2 - x s_0)/M^2} + O(1/Q^4), \quad (43)$$

where $\Phi_x^Q(1,\xi,W^2) \equiv (\mathrm{d}/\mathrm{d}x)\Phi^Q(x,\xi,W)|_{x=1}$.

As has been discussed at length in the literature, (43) can be interpreted as the so-called "end-point" contribution which arises from large transverse dinstances between two photons in the hard block [18,27]. In general, the 2π distribution amplitude $\Phi^Q(x,\xi,W^2)$ depends also on the factorization scale μ which separates large and short distances. As $x \sim 1$ the quark virtuality (the magnitude of the quark denominator in (31)) is of order of the Borel mass M. Therefore the two-pion DA in (43) should be evaluated at a low normalization point, of order of M.

In Fig. 1 we show an average value of the momentum fraction x in the integral (42) calculated as a function of Q^2. One observes that the mean value of x in (42) is indeed close to 1.

As $Q^2 \to \infty$ the second term in the RHS of (42) gives

$$\int_0^{x_0} \mathrm{d}x \frac{2x}{1-x} \Phi^Q(x,\xi,W^2) = \int_0^1 \mathrm{d}x \frac{2x}{1-x} \Phi^Q(x,\xi,W^2) + O(1/Q^2). \quad (44)$$

It reproduces the leading-order, leading-twist factorization formula when $q'^2 = 0$ in (31) and provides correct asymptotics for very large Q^2. The power correction is suppressed as $1/Q^2$.

The LO sum rule (42) results in an expression for the amplitude $T^{\gamma\pi\pi}$ which includes contributions from both the "end-point" region $x \sim 1$, associated with large transverse distances, and from small transverse distances where the $q\bar{q}$ pair is created by two photons in a compact configuration. From the sum rule it follows that the "end-point" contribution is suppressed by $1/Q^2$ as compared with the LO factorization result, i.e. it has the same order as factorizable higher-twist corrections. Despite formal power suppression, the "end-point" contribution can be numerically important for realistic values of Q^2. Our numerical analysis suggests that the sum rule (42) can be applied for the description of the amplitude $T^{\gamma\pi\pi}$ starting from moderate momentum transfers $Q^2 \geq 1\,\mathrm{GeV}^2$.

5 Radiative corrections

In principle, the sum rule (42) can be improved in a twofold way: by including the NLO α_s and higher-twist corrections to the spectral density (33). In this section we consider the NLO contribution. Taking into account higher twists requires knowledge of the corresponding two-pion distribution amplitudes which are not known yet.

One-loop corrections for the real photon case have been considered in [19]. In the current situation one should calculate the coefficient functions in the kinematics when both photons are virtual. One should also keep in mind that at the NLO diagrams with gluons enter the game. As in [19] one can use crossing symmetry to derive coefficient functions from the corresponding coefficient functions in the DVCS kinematics, as computed in [28–30]. The calculation is straightforward. The NLO amplitude can be written in the standard form:

$$T_0^{\gamma^*\pi\pi}(Q,q',\xi,W) = \sum_q e_q^2 \left(\int_0^1 \mathrm{d}x \Phi^Q(x,\xi,W^2) \times \left[C_q^0(x,\omega) + \tfrac{\alpha_\mathrm{S}(\mu^2)}{4\pi} C_q^1(x,\omega,\mu)\right] - \int_0^1 \mathrm{d}x \Phi^G(x,\xi,W^2) \left[\tfrac{\alpha_\mathrm{S}(\mu^2)}{4\pi} C_g^1(z,\omega,\mu)\right]\right). \quad (45)$$

The coefficient functions C_q^0, C_q^1 and C_g^1 are collected in the Appendix.

As in the LO case, the virtuality q'^2 enters only through the variable ω. It is convenient to rewrite (45) in the form which resembles the structure of the LO answer (31):

$$T_0^{\gamma^*\pi\pi}(Q,q',\xi,W) = \int_0^1 \mathrm{d}x \frac{x\rho(x,\xi,W^2)}{1-x\omega}, \quad (46)$$

with

$$\rho(x, \xi, W^2) = \rho_0(x, \xi, W^2) + \frac{\alpha_S(\mu^2)}{4\pi}\rho_1(x, \xi, W^2). \quad (47)$$

As in (31)

$$\rho_0(x, \xi, W^2) = 2\left(\sum e_q^2\right)\Phi^Q(x, \xi, W^2). \quad (48)$$

The NLO correction $\rho_1(x, \xi, W^2)$ can be obtained from the corresponding coefficient functions (45):

$$\sum e_q^2 \int_0^1 \mathrm{d}x \left[\Phi^Q(x, \xi, W^2)\frac{1}{\pi}\mathrm{Im}_{q'^2} C_q^1(x, \omega) - \Phi^G(x, \xi, W^2)\frac{1}{\pi}\mathrm{Im}_{q'^2} C_g^1(x, \omega)\right] = u^2 \rho_1(u, \xi, W^2)|_{u=1/\omega}, \quad (49)$$

where $\mathrm{Im}_{q'^2}$ denotes the imaginary part with respect to the variable q'^2, considered here as a complex variable with positive real as well as (infinitesimal) imaginary parts.

With such a definition the structure of the NLO LCSR for the amplitude $T_0^{\gamma\pi\pi}$ remains the same as the LO one:

$$T_0^{\gamma\pi\pi}(Q, \xi, W) = \frac{Q^2}{m_\rho^2}\int_{x_0}^1 \mathrm{d}x \rho(x, \xi, W^2) \times \exp\left\{\frac{x m_\rho^2 - \bar{x}Q^2}{xM^2}\right\} + \int_0^{x_0} \mathrm{d}x \frac{x}{1-x}\rho(x, \xi, W^2). \quad (50)$$

As in the case of the LO sum rule (42), as $Q^2 \to \infty$ (50) reproduces the perturbative expansion of the $T_0^{\gamma\pi\pi}$ amplitude, including the NLO corrections [19]. The power correction is suppressed as $1/Q^2$.

As an illustration, we quote now the form of $\rho_1(x, \xi, W^2)$ in the case of the "minimal model" (20):

$$\begin{aligned}\rho_1(x, \xi, W^2) &= \sum e_q^2 B(\xi, W^2)\left\{\ln(\mu^2/Q^2)\rho_{10}(x, \mu^2) + \rho_{11}(x, \mu^2)\right\},\\ \rho_{10}(x, \mu^2) &= (-40)x(1-x)(2x-1)R(\mu^2),\\ \rho_{11}(x, \mu^2) &= \rho_{\mathrm{as}}(x) + R(\mu^2)[\rho_{\mathrm{as}}(x) + D(x)],\end{aligned} \quad (51)$$

where $R(\mu^2)$ is defined in (21). The function $D(x)$ is a shorthand notation for

$$D(x) = -\frac{10}{3}[2\bar{x}(1-6x) + x\bar{x}(2x-1)(31 + 12\ln(x/\bar{x}))]. \quad (52)$$

Taking for M^Q the asymptotic value (17) results in $R(\mu^2 = \infty) = 0$ and

$$\rho_1(x, \xi, W^2) = B(\xi, W^2)\rho_{\mathrm{as}}(x), \quad (53)$$

where

$$\rho_{\mathrm{as}}(x) = \frac{-20C_F}{N_f + 4C_F}x(1-x)(2x-1) \times (\pi^2 - 17 - 3\ln^2(x/\bar{x})). \quad (54)$$

In this case ρ_1 has no dependence on μ.

Before evaluating the sum rule one has to provide an estimate of the factorization scale μ. This is a standard problem in a calculation based on the fixed-order perturbation theory. Note that the scale of the perturbative expansion $\mu^2 \sim Q^2$ is different from the characteristic scale of soft, "end-point" contributions $\mu^2 \sim M^2$. As a consequence, the correct treatment of the sum rule requires applying various normalization scales to various terms in (50). To avoid confusion, we propose the following procedure. One separates the perturbative contribution and writes the final formula in the form

$$T_0^{\gamma\pi\pi}(Q, \xi, W) = T_0^{\mathrm{pert}}(Q, \xi, W) + T_0^{\text{non-pert}}(Q, \xi, W). \quad (55)$$

T_0^{pert} is the perturbative amplitude (6), which can be represented as

$$T_0^{\mathrm{pert}}(Q, \xi, W) = \int_0^1 \mathrm{d}x \frac{x}{1-x}\rho(x, \xi, W^2). \quad (56)$$

The scale in this term is set by the hard photon virtuality Q^2. The second term should, as a matter of fact, be considered as the proper light-cone sum rule result:

$$T_0^{\text{non-pert}}(Q, \xi, W) = \int_{x_0}^1 \mathrm{d}x \rho(x, \xi, W^2) \times \left[\frac{Q^2}{m_\rho^2}\exp\left\{\frac{x m_\rho^2 - \bar{x}Q^2}{xM^2}\right\} - \frac{x}{1-x}\right]. \quad (57)$$

Here the integration region is restricted to $x_0 \le x \le 1$. As discussed in the previous section, this term should be evaluated at a *low* normalization point $\sim M^2$.

Of course, when evaluated at the same normalization scale, the sum of (56) and (57) reproduces the original sum rule (50).

6 Numerical results

In the subsequent numerical analysis the following input has been used: the threshold parameter $s_0 = 1.5\,\mathrm{GeV}^2$ has been taken from the two-point sum rule [31]. This sum rule is reliable for the corresponding Borel parameter $M_{2\mathrm{pt}}^2 = 0.5$–$0.8\,\mathrm{GeV}^2$. In the light-cone sum rules it should be larger to compensate for the fact that the effective expansion parameter is given by the inverse power of xM^2. In this case a typical choice is $M^2 \sim M_{2\mathrm{pt}}^2/\langle x\rangle$ [17, 18]. We assume that $0.6 \le M^2 \le 1.2\,\mathrm{GeV}^2$ is a reasonable interval. In addition we have checked that changing s_0 by $\pm 0.2\,\mathrm{GeV}^2$ does not produce any sizable effect.

We remind the reader that in the present investigation we have neglected higher-twist contributions to the sum rule for $T_0^{\gamma^*\pi\pi}$. They are, as usual, suppressed by additional powers of the Borel parameter. To obtain their contribution one should know two-pion distribution amplitudes of higher twists which have not yet been studied. In the case of photon–pion transition form factor [17],

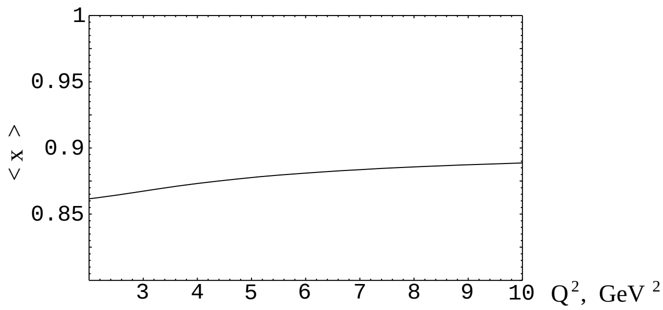

Fig. 2. Average momentum fraction x as a function Q^2. We take $M^2 = 0.9\,\mathrm{GeV}^2$ and $s_0 = 1.5\,\mathrm{GeV}^2$; see the explanation in the text

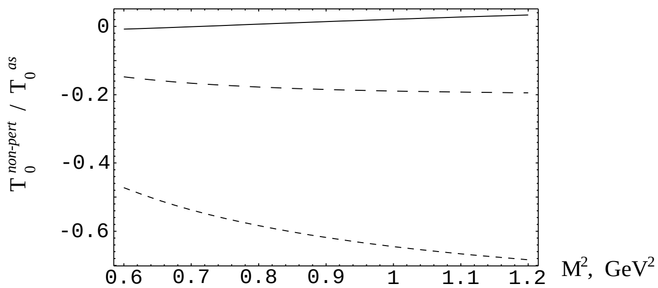

Fig. 3. Ratio $T_0^{\text{non-pert}}/T_0^{\text{as}}$ as a function of M^2 for $Q^2 = 1\,\mathrm{GeV}^2$ (short-dashed), $Q^2 = 3\,\mathrm{GeV}^2$ (long-dashed) and $Q^2 = 10\,\mathrm{GeV}^2$ (solid line)

the contribution of such terms to the non-perturbative (power-suppressed) part is about 30–35% of the leading-twist contribution for the low $Q^2 \sim 1$–$2\,\mathrm{GeV}^2$. Qualitatively, the same picture should hold also in the present case.

All numerical results have been obtained with the model (20) for the 2π distribution amplitude. We have used the quark momentum fraction $M^Q(0.6\,\mathrm{GeV}^2) = 0.63$ taken from the GRV parametrization [26]. We have kept $N_f = 4$ which results in $M^Q_{\text{as}} = 0.43$.

In all calculations we have used $\Lambda_{\text{QCD}} = 0.204\,\mathrm{GeV}$ and one-loop running coupling $\alpha_{\mathrm{S}}(1\,\mathrm{GeV}^2) = 0.47$.

In the numerical analysis one has to specify the factorization scale. Since after subtraction of the perturbative amplitude $T_0^{\text{non-pert}}(Q, \xi, W)$ is dominated by the "end-point", soft contributions, we apply here a fixed, low scale $\mu^2 \sim M^2$.

To estimate the ambiguity due to the scale dependence in $T_0^{\text{pert}}(Q, \xi, W)$ we parameterize μ^2 according to

$$\mu^2 = \kappa Q^2 + M^2, \tag{58}$$

with a parameter κ which varies in the interval $1/5 \leq \kappa \leq 1$.

We consider values of Q^2 in the physically interesting interval $1\,\mathrm{GeV}^2 \leq Q^2 \leq 10\,\mathrm{GeV}^2$.

To cancel the influence of the overall factor $B(\xi, W^2)$, in the following we always plot the ratio of $T_0^{\gamma\pi\pi}(Q^2, \xi, W^2)$ to its asymptotic value at $Q^2 = \infty$ (23).

The Borel parameter dependence of the ratio $T_0^{\text{non-pert}}/T_0^{\text{as}}$ is shown in Fig. 3. We find that the M^2 dependence is

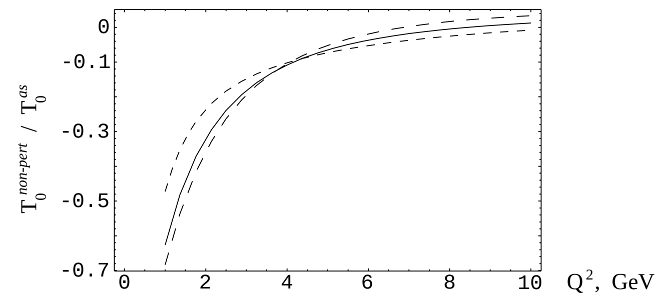

Fig. 4. Ratio $T_0^{\text{non-pert}}/T_0^{\text{as}}$ as a function of Q^2. Short- and long-dashed lines correspond to $M^2 = 0.6$ and $1.2\,\mathrm{GeV}^2$, with $\mu^2 = M^2$, respectively. The solid line ($M^2 = 0.9\,\mathrm{GeV}^2$) represents the parametrization (59)

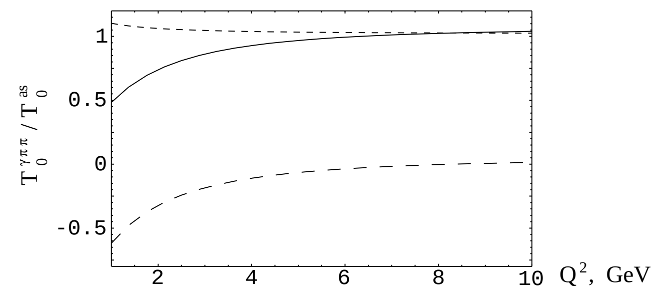

Fig. 5. The ratio $T_0^{\gamma\pi\pi}/T_0^{\text{as}}$ as a function of Q^2 (solid line). Short-dashed line represents $T_0^{\gamma\pi\pi}/T_0^{\text{as}}$ with $\mu^2 = 2/5Q^2 + 1\,\mathrm{GeV}^2$. The long-dashed line represents $T_0^{\text{non-pert}}/T_0^{\text{as}}$ with $\mu^2 = M^2 = 0.9\,\mathrm{GeV}^2$. The solid line is the sum of both

sufficiently flat to justify the use of LCSRs, although for lower values of Q^2, where power corrections are increasingly important, the dependence on the Borel parameter becomes stronger. Assuming that the uncertainty arising from the Borel parameter dependence should not exceed 30% for the $T_0^{\text{non-pert}}$, we estimate that the sum rule (42) provides a reasonable description of $T_0^{\gamma\pi\pi}$ starting from $Q^2 \geq 1\,\mathrm{GeV}^2$.

In Fig. 4 we show the ratio $T_0^{\text{non-pert}}(Q^2)/T_0^{\text{as}}$ obtained from the light-cone sum rule (57) for different values of the Borel parameter M^2. One observes that in the whole interval of Q^2 considered here the sum rule calculation is rather stable with respect to variation of the Borel parameter within a reasonable interval. In accordance with expectations, the non-perturbative correction becomes smaller with increasing Q^2. By fitting a simple formula we found that for the value of the Borel parameter in the middle of the interval, $M^2 = 0.9\,\mathrm{GeV}^2$, and in the Q^2 region $1 \leq Q^2 \leq 10\,\mathrm{GeV}^2$, $T_0^{\text{non-pert}}/T_0^{\text{as}}$ can be parameterized as

$$T_0^{\text{non-pert}}(Q^2)/T_0^{\text{as}} = \frac{-1.5 + 0.05Q^2 + 0.015Q^4}{1 + Q^2 + 0.3Q^4}, \tag{59}$$

with Q^2 in units of GeV^2; see the solid line in Fig. 4.

In Fig. 5 we present perturbative and non-perturbative contributions to the ratio $T_0^{\gamma\pi\pi}/T_0^{\text{as}}$ as a function of Q^2. $T_0^{\text{non-pert}}$ is calculated with $M^2 = 0.9\,\mathrm{GeV}^2$. The non-perturbative corrections is numerically significant only in the region $Q^2 \leq 4\,\mathrm{GeV}^2$. For higher values of Q^2 the amplitude is dominated by the NLO leading-twist contribution.

7 Summary and conclusions

The main result of this paper is the numerical estimate of the power-suppressed correction to the leading-twist helicity-conserving amplitude of the process $\gamma^*\gamma \to \pi\pi$. The light-cone sum rules technique allows one to circumvent difficulties due to non-factorizability of the power-suppressed terms. Although formally our analysis is not complete, as we have neglected contribution of higher-twist operators to the amplitude of two-pion production in a collision of two virtual photons, we believe that the general picture is reliable, at least qualitatively. Power corrections are increasingly important with decreasing Q^2 for $Q^2 \leq 4\,\mathrm{GeV}^2$, and become about 50% of the leading-twist amplitude at $Q^2 = 1\,\mathrm{GeV}^2$.

Our final result for the helicity-conserving $\gamma^*\gamma \to \pi\pi$ amplitude is shown in Fig. 6. The grey bound indicates uncertainty due to the linearly combined Borel parameter and the factorization scale variations. One finds that starting from Q^2 around $4\,\mathrm{GeV}^2$ the twist-2 contribution approximately saturates the amplitude. This observation suggests that the cross-section of the process $\gamma^*\gamma \to \pi\pi$ can be accurately predicted in QCD for given models of 2π distribution amplitudes.

Assuming dominance of the lowest conformal wave, we have found that the helicity-conserving amplitude is very sensitive to the normalization of the gluonic 2π distribution amplitude. This observation, combined with crossing, makes it plausible to use $\gamma^*\gamma \to \pi\pi$ to constrain the momentum fraction carried by gluons in a pion.

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Appendix

A Coefficient functions

To the NLO accuracy the amplitude $T_0^{\gamma^*\pi\pi}$ can be represented as a standard convolution of 2π distribution amplitudes with the hard scattering coefficients:

$$T_0^{\gamma^*\pi\pi}(Q,q',\xi,W) = \sum_q e_q^2 \left(\int_0^1 \mathrm{d}x \Phi^Q(x,\xi,W^2) \right.$$
$$\times \left[C_q^0(x,\omega) + \frac{\alpha_S(\mu^2)}{4\pi} C_q^1(x,\omega,\mu^2/Q^2) \right]$$
$$\left. - \int_0^1 \mathrm{d}x \Phi^G(x,\xi,W^2) \left[\frac{\alpha_S(\mu^2)}{4\pi} C_g^1(z,\omega,\mu^2/Q^2) \right] \right), \tag{A.1}$$

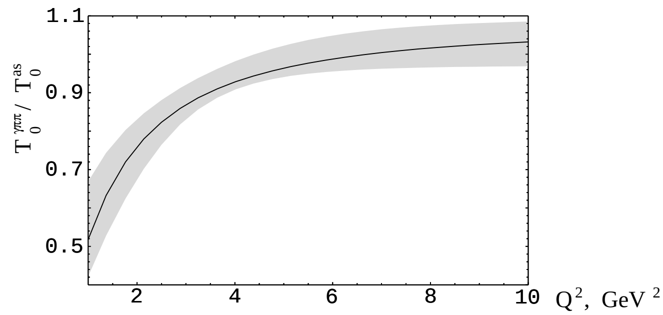

Fig. 6. The LCSR prediction for the ratio $T_0^{\gamma\pi\pi}/T_0^{\mathrm{as}}$ as a function of Q^2. The grey band shows the sensitivity of our result to a variation of the Borel parameter within $0.6 \leq M^2 \leq 1.2\,\mathrm{GeV}^2$ and the factorization scale according to formula (58)

where ω has been defined in (29). The coefficient functions in the $\overline{\mathrm{MS}}$ scheme read

$$C_q^0(x,\omega) = \frac{2x}{1-x\omega}, \tag{A.2}$$

$$C_q^1(x,\omega,\mu^2/Q^2) = C_F \left[\ln(\mu^2/Q^2)[C_q^{10}(x,\omega) \right.$$
$$\left. -C_q^{10}(1-x,\omega)] + C_q^{11}(x,\omega) - C_q^{11}(1-x,\omega) \right],$$

$$C_q^{10}(x,\omega) = -\frac{3}{(1-x\omega)\omega} - \ln(1-\omega)\frac{2(1-\omega)}{\bar{x}\omega^2}$$
$$\times \left(\frac{1}{1-x\omega} - \frac{1}{\omega} \right) + 2\frac{(1-\omega)}{1-x\omega}\frac{\ln(1-x\omega)}{\bar{x}\omega^2}$$
$$+2\frac{\ln(1-x\omega)}{x\bar{x}\omega^2}\left[\frac{1-x\omega}{\omega} - \frac{\bar{x}}{1-x\omega} \right], \tag{A.3}$$

$$C_q^{11}(x,\omega) = \frac{1}{(1-x\omega)\omega}\left[-9 + \frac{2}{x\omega}\ln^2(1-x\omega) \right.$$
$$\left. -\frac{3}{x\omega}\ln(1-x\omega) + \frac{3}{x\omega}\ln(1-\omega) - \frac{1}{x\omega}\ln^2(1-\omega) \right]$$
$$+\frac{3}{\bar{x}\omega^2}\ln(1-x\omega) - \frac{3}{x\bar{x}\omega^2}\ln(1-\omega)$$
$$+(x-1-1/\omega)\frac{\ln^2(1-x\omega)}{x\bar{x}\omega^2}$$
$$+(1-x+x/\omega)\frac{\ln^2(1-\omega)}{x\bar{x}\omega^2}, \tag{A.4}$$

$$C_g^1(x,\omega,\mu^2/Q^2) = \ln(\mu^2/Q^2) C_g^{10}(x,\omega) + C_g^{11}(x,\omega), \tag{A.5}$$

$$C_g^{10}(x,\omega) = \frac{(-2)}{w^4(x\bar{x})^2}$$
$$\times \left\{ (1-\omega+[1-x\omega]^2)\ln(1-x\omega) \right.$$
$$+(1-\omega+[1-\bar{x}\omega]^2)\ln(1-\bar{x}\omega)$$
$$\left. -(2-\omega x^2-\omega\bar{x}^2)(1-\omega)\ln(1-\omega) \right\}, \tag{A.6}$$

$$C_g^{11}(x,\omega) = \frac{1}{w^4(x\bar{x})^2}\left\{ a_1(x,\omega)\ln(1-\omega) \right.$$
$$+a_2(x,\omega)\ln^2(1-\omega) + c_1(x,\omega)\ln(1-x\omega)$$

$$
\begin{aligned}
&+c_1(\bar{x},\omega)\ln(1-\bar{x}\omega) + c_2(x,\omega)\ln^2(1-x\omega)\\
&+ c_2(\bar{x},\omega)\ln^2(1-\bar{x}\omega)\big\}\,,\\
&a_1(x,\omega) = 8 + 4\omega(x-3-x^2) + 4\omega^2(1-x+x^2),\\
&a_2(x,\omega) = -2 + \omega(3-2x+2x^2) - \omega^2(1-2x+2x^2),\\
&c_1(x,\omega) = -8 + 4\omega(1+2x) - 2\omega^2 x(1+x),\\
&c_2(x,\omega) = 2 - \omega(1+2x) + \omega^2 x^2. \qquad \text{(A.7)}
\end{aligned}
$$

Here we used the shorthand notation $\bar{x} \equiv 1-x$. Note that the physical amplitude does not have a singularity (pole) when $\omega \to 0$ and therefore all coefficient functions must be well defined in this limit:

$$
\begin{aligned}
&C_q^1(x,\omega,\mu^2/Q^2) = C_F\left[\ln(\mu^2/Q^2)\frac{8}{3}(2x-1)\right.\\
&\left.+ (1-2x)\right] + O(\omega), \qquad \text{(A.8)}\\
&C_g^1(x,\omega,\mu^2/Q^2) = \ln(\mu^2/Q^2)\frac{4}{3} + \frac{7}{3} + O(\omega).
\end{aligned}
$$

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